Note that the logarithmic and power-law profiles of the velocity do not satisfy the physical condition $d\bar{v}/dr = 0$ at $r = 0$ [1, 3]. This deficiency is eliminated here.

Thus, the function in Eq. (3) leads to a result in agreement with the well-known laws of turbulent friction.

NOTATION

R, l, radius and length of the channel; r, z, cylindrical coordinates, referred to R and l, respectively; v, gas velocity; v_0, gas velocity at the channel axis; $U = v/v_0$; C_f, friction coefficient.

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TWO-DIMENSIONAL FLOW OF VISCOUS FLUID BETWEEN CYLINDRICAL ROLLERS ROTATING IN OPPOSITE DIRECTIONS

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A method of calculating the hydrodynamic parameters of two-dimensional flow of a viscous fluid through a channel formed by rotating cylinders is described.

An important role in the reprocessing of polymer materials in rolling machines (cylinders, calenders) is played by the flow of the viscous fluid through the gap between oppositely rotating cylindrical rollers. The polymer between rollers is usually in the molten state, characterized by complex hydrodynamic effects which influence the quality of the end product. These effects include a "rotating stock" of material within the deformation zone, which appears when the equipment is heavily loaded. In order to quantitatively estimate the flow characteristics under given technological conditions in a given equipment, it is necessary to mathematically describe the process and to construct an algorithm for calculating the distribution of the velocity components at which particles of the material move through the deformation zone as well as the integral characteristics related to this distribution.

The flow region of the material is shown schematically in Fig. 1.

In order to describe the process theoretically, it is necessary to solve the complete system of two-dimensional Navier–Stokes equations with uniqueness conditions stipulated for the given curvilinear channel.

It is well known that polymer materials are mostly nonlinearly viscous media and, therefore, the Newton hypothesis of friction is inadequate for their description, which limits the practical use of the method which will be described here. We will nevertheless assume first that the material to be reprocessed is Newtonian fluid, the purpose being to simplify the development of this method of calculating a two-dimensional flow through a curvilinear channel and facilitating the test calculations. It will be assumed, furthermore, that the flow is steady and isothermal, also that the rate of material processing is held constant by the feeder band (Fig. 1a) moving around a roller until it enters the gap between rollers as a solid body at the same angular velocity. The flow region is bounded by the rollers on its left-hand and right-hand sides and by the exit coordinate from below, this coordinate being easily calculated from the given entrance coordinate by well-known methods such as those, for instance, which use the condition of constant material flow rate [1].

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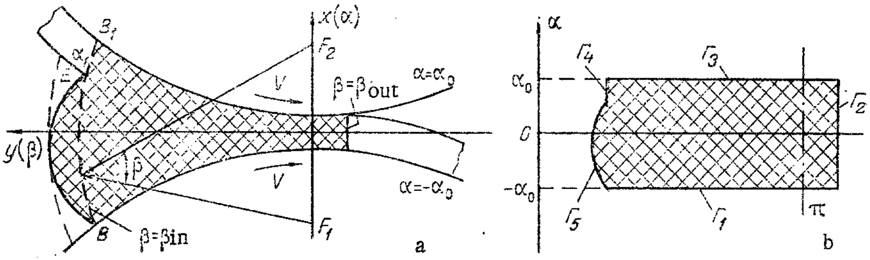

Fig. 1. Schematic diagram of flow region of material between rollers: (a) in Cartesian system of coordinates; (b) in bipolar system of coordinates.

Inasmuch as we are interested in a process with large stock of material in the deformation zone, i.e., a process with a counterflow zone and closed streamlines (corresponding to $\xi_H \geqslant 3$) we can use the results of earlier studies [2, 3] and assume that to various values of the entrance coordinate ξ_H corresponds a single maximally possible exit coordinate $\xi_{1,max}$. For any arbitrarily stipulated entrance coordinate, accordingly, the corresponding exit coordinate will be always known.

We will now analyze the process in a bipolar system of coordinates defined by the relations

$$\alpha = \ln \frac{r_1}{r_2}, \quad \beta = \theta_1 - \theta_2, \tag{1}$$

$$x = \frac{a \sin \beta}{\operatorname{ch} \alpha - \cos \beta}, \quad y = \frac{a \operatorname{sh} \alpha}{\operatorname{ch} \alpha - \cos \beta}. \tag{2}$$

Here the coordinate of material entrance into the deformation zone corresponds to some coordinate line, namely a circle β = const on which P = 0 by the definition of entrance. The boundary of the flow region will be assumed to consist of two parts, material being fed only across the part $\beta = \beta_{in}$ which corresponds to the thickness of the feeder band from point E to point B_1. The other part of the boundary, from point E (with coordinates α_1, β_{in}) to point $B(-\alpha_0, \beta_{in})$ is a free surface, free of normal and shearing stresses, but its shape is not a priori known. We will, therefore, seek the shape of this free surface in the course of solving the problem. First, however, its shape can be stipulated in the form of some curve on the basis of experimental data or a priori information about the process. The lower boundary of the flow region coincides with the $\beta = \beta_{out}$ part of the coordinate line between the moving walls $\alpha = \alpha_0$ and $\alpha = -\alpha_0$. In the bipolar system of coordinates the closed two-dimensional region of flow will have rectilinear boundaries, except along the free surface (Fig. 1b). Boundaries Γ_1 and Γ_3 correspond to the moving walls, Γ_2 corresponds to the exit coordinate, Γ_4 corresponds to the feeder band, and Γ_5 corresponds to the free surface. Two-dimensional isothermal steady flow within this closed region, with small values of the Reynolds number ($\mathrm{Re} \approx 10^{-3}$), is described by the system of equations of motion and continuity. In the Cartesian system of coordinates this system of equations is

$$\frac{\partial P}{\partial x} = \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \tag{3}$$

$$\frac{\partial P}{\partial y} = \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{5}$$

Upon introduction of the flow functions $\psi(x, y)$, letting

$$u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x}, \tag{6}$$

Eqs. (3)-(5) transform into

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial y} (\mu \Delta \psi), \tag{7}$$

$$\frac{\partial P}{\partial y} = -\frac{\partial}{\partial x} (\mu \Delta \psi). \tag{8}$$

Elimination of the pressure yields the biharmonic equation

$$\Delta\Delta\psi(x, y) = 0 \tag{9}$$

for the flow function. The flow function $\psi(x, y)$ found from this equation has the property that its isolevel lines $\psi(x, y)$ = const are also streamlines, which makes it easy to reconstruct the flow pattern from the calculated distribution of this function. In the bipolar system of coordinates Eq. (9) becomes

$$\frac{1}{h^2(\alpha, \beta)}\,\Delta\left[\frac{1}{h^2(\alpha, \beta)}\,\Delta\psi(\alpha, \beta)\right] = 0. \tag{10}$$

In order to ensure uniqueness of the solution to the biharmonic equation, it is necessary and sufficient to impose two constraints on function ψ and its derivatives at the boundary of the closed region. The boundary conditions for each of its five parts have their singularities. At the boundary Γ_1, where $\alpha = -\alpha_0$,

$$\psi(-\alpha_0, \beta) = 0 \quad (\beta_{in} \leqslant \beta \leqslant \beta_{out}). \tag{11}$$

This condition signifies that Γ_1 is a streamline, which corresponds to the condition of material adhesion to the roller surface. The second condition is motion of a wall at velocity V

$$\frac{\partial\psi}{\partial\alpha}(-\alpha_0, \beta) = -Vh(-\alpha_0, \beta) \quad (\beta_{in} \leqslant \beta \leqslant \beta_{out}). \tag{12}$$

At the boundary Γ_2 one can assume that the u-component of velocity is zero and the v-component of velocity is uniform

$$\psi(\alpha, \beta_{out}) = \int_{-\alpha_0}^{\alpha} Vh(\alpha, \beta)\,d\alpha = \Phi(\alpha) \quad (-\alpha_0 \leqslant \alpha \leqslant \alpha_0). \tag{13}$$

It follows from u = 0 that

$$\frac{\partial\psi}{\partial\beta}(\alpha, \beta_{out}) = 0 \quad (-\alpha_0 \leqslant \alpha \leqslant \alpha_0). \tag{14}$$

Boundary Γ_3 corresponds to the upper wall. The condition for adhesion here must be $\psi(\alpha_0, \beta)$ = const, where the value of this constant must match the velocity profile at the entrance boundary Γ_4 so that function ψ will not be discontinuous at the corner point

$$\psi(\alpha_0, \beta) = \Phi(\alpha_0). \tag{15}$$

The condition for motion of the wall at a linear velocity V is

$$\frac{\partial\psi}{\partial\alpha}(\alpha_0, \beta) = -Vh(\alpha_0, \beta) \quad (\beta_{in} \leqslant \beta \leqslant \beta_{out}). \tag{16}$$

At the boundary Γ_4, we proceed as we did at the boundary Γ_2, namely

$$\psi(\alpha, \beta_{in}) = \int_{-\alpha_1}^{\alpha} Vh(\alpha, \beta_{in})\,d\alpha = \Phi_1(\alpha) \quad (\alpha_1 \leqslant \alpha \leqslant \alpha_0). \tag{17}$$

The condition for motion of point $E(\alpha_1, \beta_{in})$ at a linear velocity V_1 is

$$\frac{\partial\psi}{\partial\alpha}(\alpha, \beta_{in}) = 0 \quad (\alpha_1 \leqslant \alpha \leqslant \alpha_0). \tag{18}$$

Most intricate is formulation of the boundary conditions at the free surface Γ_5. The conventional condition for a free surface of a viscous fluid, used by many authors [4-6], is a zero stress tensor applied to the vector of normals $T \cdot n = 0$. The physical meaning of this condition is a normal stress equal to the external pressure and absence of shearing stress. The differential form of the stress conditions at the free surface is

$$-2\mu\frac{\partial u_n}{\partial n} + P = 0, \tag{19}$$

$$\frac{\partial u_n}{\partial s} + \frac{\partial u_s}{\partial n} = 0. \tag{20}$$

Since the true shape of the free surface is not known, one must stipulate for it some likely approximation. For finding the true shape of the free surface one needs one more condition. At this third condition for the free surface we use

$$\psi(\alpha, \beta)=0, \tag{21}$$

where $\beta=\varphi(\alpha)$ describes the free surface. This condition implies that the free surface is a streamline. Its physical meaning can be interpreted as follows. We stipulate some shape of boundary Γ_5 and imagine a stationary solid wall placed on it. We then solve the problem for one of the conditions, say condition (20), and determine the flow pattern. We then check condition (19). When the latter is satisfied, then the stipulated shape of boundary Γ_5 indeed approximates the free surface (the wall can now be removed and the shape of the boundary will not change). When condition (19) is not satisfied, then we change the shape of boundary Γ_5 and thus also of the stationary wall and again solve the problem, but this time for conditions (20) and (21). We proceed in this manner until we obtain the true shape of the free surface. For solution of this problem, we express conditions (19) and (20) in terms of function ψ. Condition (20) for the shearing stresses becomes

$$\frac{\partial}{\partial s}\left(\frac{1}{h}\frac{\partial\psi}{\partial s}\right)=\frac{\partial}{\partial n}\left(\frac{1}{h}\frac{\partial\psi}{\partial n}\right). \tag{22}$$

According to condition (21), $\psi(\alpha, \beta) = 0$ at the boundary Γ_5 and thus ψ = const on the line $\beta = \varphi(\alpha)$ so that $\partial\psi/\partial s = 0$, from which follows

$$\frac{\partial}{\partial s}\left(\frac{1}{h}\frac{\partial\psi}{\partial s}\right)=0$$

and consequently

$$\frac{\partial}{\partial n}\left(\frac{1}{h}\frac{\partial\psi}{\partial n}\right)=0. \tag{23}$$

Condition (19) for the normal stresses in dimensionless form becomes

$$P=\frac{2}{\mathrm{Re}}\frac{1}{h}\frac{\partial u_n}{\partial n}. \tag{24}$$

Since $u_n=\frac{1}{h}\frac{\partial\psi}{\partial s}$, hence

$$P=\frac{2}{\mathrm{Re}}\frac{1}{h}\frac{\partial}{\partial n}\left(\frac{1}{h}\frac{\partial\psi}{\partial s}\right). \tag{25}$$

On the other hand, the Navier–Stokes equations yield

$$\frac{\partial P}{\partial\alpha}=\mu\frac{\partial}{\partial\beta}\left(\frac{1}{h^2}\Delta\psi\right), \tag{26}$$

$$\frac{\partial P}{\partial\beta}=\mu\frac{\partial}{\partial\alpha}\left(\frac{1}{h^2}\Delta\psi\right). \tag{27}$$

From these expressions one can easily obtain, in dimensionless form, the relation

$$\frac{\partial P}{\partial s}=\frac{1}{\mathrm{Re}}\frac{\partial}{\partial n}\left(\frac{1}{h^2}\Delta\psi\right). \tag{28}$$

to be satisfied at the free surface. Differentiating expression (25) with respect to s yields

$$\frac{\partial P}{\partial s}=\frac{2}{\mathrm{Re}}\frac{\partial}{\partial s}\left[\frac{1}{h}\frac{\partial}{\partial n}\left(\frac{1}{h}\frac{\partial\psi}{\partial s}\right)\right]. \tag{29}$$

From Eqs. (28) and (29) we obtain a relation for checking the shape of the free surface, namely

$$\frac{\partial}{\partial n}\left(\frac{1}{h^2}\Delta\psi\right)=\frac{\partial}{\partial s}\left[\frac{1}{h}\frac{\partial}{\partial n}\left(\frac{1}{h}\frac{\partial\psi}{\partial s}\right)\right]. \tag{30}$$

We now introduce the velocity vortex

$$\zeta=\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x} \tag{31}$$

or, in bipolar coordinates,

$$\zeta=\frac{1}{h^2}\left\{\frac{\partial(uh)}{\partial\beta}-\frac{\partial(vh)}{\partial\alpha}\right\}. \tag{32}$$

Considering that

$$v = -\frac{1}{h}\frac{\partial\psi}{\partial\alpha}, \quad u = \frac{1}{h}\frac{\partial\psi}{\partial\beta},$$

we find that

$$\zeta = \frac{1}{h^2}\Delta\psi. \tag{33}$$

The biharmonic equation is then equivalent to the system Laplace and Poisson equations

$$\Delta\zeta = 0, \tag{34}$$

$$\Delta\psi = h^2\zeta. \tag{35}$$

The expression for the pressure gradient can now be rewritten in terms of

$$\frac{\partial P}{\partial\alpha} = \mu\frac{\partial\zeta}{\partial\beta}, \tag{36}$$

$$\frac{\partial P}{\partial\beta} = -\mu\frac{\partial\zeta}{\partial\alpha}. \tag{37}$$

For solving this problem numerically, the flow region is subdivided by a grid of β = const and α = const coordinate lines. In the bipolar system of coordinates this grid transforms into a nonuniform rectangular one. The shape of the free surface is initially stipulated in Cartesian coordinates and then transcribed into the corresponding $\beta = \varphi(\alpha)$ curve along the boundary Γ_5 in bipolar coordinates. In order to ensure uniqueness of the solution to the system of Eqs. (34), (35), it is necessary to stipulate two conditions at the boundary: a constraint on function ζ and a constraint on function ψ. The physical meaning of the process provides a simple guideline for imposing the constraint on function ψ. The constraint on function ζ is stipulated approximately, on the basis of the relation between ζ and ψ. The boundary condition at the wall is

$$\psi(i, j_c) = 0. \tag{38}$$

with α-layers and β-layers of the grid denoted by indices i and j respectively, and j_C denoting the number of the coordinate line which corresponds to the boundary Γ_1.

Expanding function ψ in the vicinity of a point (i_C, j) into a Taylor series and retaining the terms of up to second order, considering also that u = 0 at the boundary Γ_1, we obtain

$$\zeta_{i,j} = \frac{2\psi_{i,j-1}}{k_j^2 h^2(i_c, j)} + \frac{2V}{h(i, j_c) k_j}, \tag{39}$$

where k_j is the α-step on the grid and $h(i, j_C) = h(\alpha_{j_C}, \beta_i)$.

At the boundary Γ_2 the first condition is the same as condition (13):

$$\psi_{i_0 j} = \Phi_{0,j}, \tag{40}$$

where i_0 is the number of the coordinate line which corresponds to the exit coordinate β_{out} (boundary Γ_2). The constraint on the vortex at the boundary Γ_2 can be obtained by expanding function ψ in the vicinity of point (0, j) into a Taylor series and considering that u = 0 and thus $\frac{\partial\psi}{\partial\beta}(i_0, j) = 0$, we obtain

$$\zeta_{0,j} = \frac{2(\psi_{1,j} - \psi_{0,j})}{h^2(0, j) h_1^2} + \frac{2}{h^2(0, j)}\left[\frac{\psi_{0,j+1}}{k_j(k_{j+1}+k_j)} - \frac{\psi_{0,j}}{k_j k_{j-1}} + \frac{\psi_{0,j-1}}{k_{j-1}(k_j+k_{j-1})}\right]. \tag{41}$$

At the entrance boundary Γ_4, by analogy, one obtains the constraints

$$\psi_{i,j} = \Phi_{N,j}, \tag{42}$$

$$\zeta_{N,j} = \frac{2(\psi_{N-1,j} - \psi_{N,j})}{h^2(N, j) h_N^2} + \frac{2}{h^2(N, j)}\left[\frac{\psi_{N,j+1}}{k_j(k_{j+1}+k_j)} - \frac{\psi_{N,j}}{k_j k_{j-1}} + \frac{\psi_{N,j-1}}{k_{j-1}(k_j+k_{j-1})}\right]. \tag{43}$$

At the boundary Γ_3 the constraint on the flow function is the same as constraint (15):

$$\psi_{i,j_c} = \Phi_{N,0}. \tag{44}$$

Using the condition of impermeability, namely u = 0, and consequently $\partial u/\partial\beta(i, 0) = 0$, we obtain

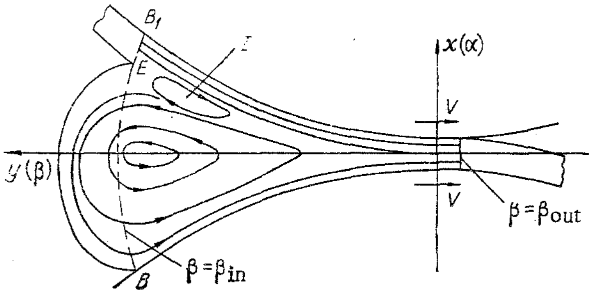

Fig. 2. Pattern of material flow through gap between rollers: region I an "entrance" vortex.

$$\zeta_{i,0} = \frac{2}{h^2(i,\ 0)\, h_1^2}\ [\psi_{i,1} + V(\Phi_{N,0} + h(i,\ 0)\, k_1)]. \tag{45}$$

At the free surface Γ_5 we have

$$\psi_{i,j} = 0. \tag{46}$$

The second constraint on function ψ is already given, namely condition (23), but it must be reformulated in terms of function

$$\frac{\partial}{\partial n}\left(\frac{1}{h}\frac{\partial\psi}{\partial n}\right) = H(\alpha,\ \beta)\frac{\partial\psi}{\partial n} + \frac{1}{h}\left(\frac{\partial^2\psi}{\partial\alpha^2}\, n_\alpha^2 + 2\,\frac{\partial^2\psi}{\partial\alpha\partial\beta}\, n_\alpha n_\beta + \frac{\partial^2\psi}{\partial\beta^2}\, n_\beta^2\right) = 0, \tag{47}$$

where $H(\alpha,\ \beta) = \frac{\partial}{\partial n}\left(\frac{1}{h(\alpha,\ \beta)}\right)$; and n_α, n_β are the corresponding vectors of normals to the free surface.

Considering that $h^2\zeta - \frac{\partial^2\psi}{\partial\alpha^2} = \frac{\partial^2\psi}{\partial\beta^2}$, we obtain from relation (33)

$$h^2 n_\beta^2\,\zeta = \frac{1}{h}\left[\frac{\partial^2\psi}{\partial\alpha^2}\,(n_\beta^2 - n_\alpha^2) - 2\,\frac{\partial^2\psi}{\partial\alpha\partial\beta}\, n_\alpha n_\beta\right] - H(\alpha,\ \beta)\left(n_\alpha\frac{\partial\psi}{\partial\alpha} + n_\beta\frac{\partial\psi}{\partial\beta}\right). \tag{48}$$

On the other hand, relation (33) yields also $h^2\zeta - \frac{\partial^2\psi}{\partial\beta^2} = \frac{\partial^2\psi}{\partial\alpha^2}$, so that

$$h^2 n_\alpha^2\,\zeta = \frac{1}{h}\left[-\frac{\partial^2\psi}{\partial\beta^2}\,(n_\beta^2 - n_\alpha^2) - 2\,\frac{\partial^2\psi}{\partial\alpha\partial\beta}\, n_\alpha n_\beta\right] - H(\alpha,\ \beta)\left(n_\alpha\frac{\partial\psi}{\partial\alpha} + n_\beta\frac{\partial\psi}{\partial\beta}\right). \tag{49}$$

Adding the symmetric conditions (48) and (49) yields

$$\zeta = \frac{1}{h^2(n_\alpha^2 + n_\beta^2)}\left\{\left(\frac{\partial^2\psi}{\partial\beta^2} - \frac{\partial^2\psi}{\partial\alpha^2}\right)(n_\alpha^2 - n_\beta^2) - 4\,\frac{\partial^2\psi}{\partial\alpha\partial\beta}\, n_\alpha n_\beta\right\} - \frac{2H(\alpha,\ \beta)}{h^2(n_\alpha^2 + n_\beta^2)}\left(\frac{\partial\psi}{\partial\alpha}\, n_\alpha + \frac{\partial\psi}{\partial\beta}\, n_\beta\right). \tag{50}$$

Finally, as the verifying condition we select the condition for the normal stresses formulated in terms of ζ:

$$\frac{\partial\zeta}{\partial n} = \frac{\partial}{\partial s}\left[\frac{1}{h}\frac{\partial}{\partial n}\left(\frac{1}{h}\frac{\partial\psi}{\partial s}\right)\right]. \tag{51}$$

The systems of Eqs. (34), (35) with boundary conditions (38)-(46) and Eqs. (50), (51) are solved by the method of successive relaxations [7]. The recurrence relations for the system of Eqs. (34), (35) at each internal node of the grid are

$$[\psi_{i,j}]^{(k+1)} = \left\{\frac{h_{i-1}h_i k_{j-1}k_j}{h_{i-1}h_i + k_{j-1}k_j}\left(\frac{[\psi_{i+1,j}]^{(k)}}{h_i(h_{i-1} + h_i)} + \frac{[\psi_{i-1,j}]^{(k+1)}}{h_{i-1}(h_{i-1} + h_i)} + \frac{[\psi_{i,j+1}]^{(k)}}{k_j(k_{j-1} + k_j)} + \frac{[\psi_{i,j-1}]^{(k+1)}}{k_{j-1}(k_{j-1} + k_j)} - \frac{1}{2}\, h_{i,j}^2\,\zeta_{i,j}\right) - [\psi_{i,j}]^{(k)}\right\}\omega + [\psi_{i,j}]^{(k)}, \tag{52}$$

$$[\zeta_{i,j}]^{(k+1)} = \left\{\frac{h_{i-1}h_i k_{j-1}k_j}{h_{i-1}h_i + k_{j-1}k_j}\left(\frac{[\zeta_{i,j+1}]^{(k)}}{h_i(h_{i-1} + h_i)} + \frac{[\zeta_{i-1,j}]^{(k+1)}}{k_{j-1}(k_{j-1} + k_j)} + \frac{[\zeta_{i,j+1}]^{(k)}}{k_j(k_{j-1} + k_j)} + \frac{[\zeta_{i,j-1}]^{(k+1)}}{k_{j-1}(k_{j-1} + k_j)}\right) - [\zeta_{i,j}]^{(k)}\right\}\omega + [\zeta_{i,j}]^{(k)}. \tag{53}$$

In these relations ω is the so-called relaxation parameters ($1 \leqslant \omega \leqslant 2$), a successful selection of its value can appreciably accelerate the convergence of the iteration process.

The method was implemented on a computer, and the qualitative flow pattern shown in Fig. 2 generalizes the results of several calculations for two-dimensional flow of a Newtonian fluid through the gap between oppositely rotating rollers. The pattern of streamlines on this diagram agrees closely with results of experimental studies [8-10] pertaining to flow of polymers through the gap between rotating rollers. A rotating stock near the free surface and an "entrance" vortex [9] are revealed by a 30 × 40 computation grid already. The distribution of velocity components u and v can be easily reconstructed from the distribution of streamlines. Determining the pressure distribution within the deformation zone requires another cycle of iteration for a solution of the Poisson equation for pressure.

Numerical studies of two-dimensional flow of a rheological power-law medium between rotating rollers are currently underway and based on this method.

NOTATION

u and v, projections of the velocity vector on the axis of coordinates; μ, dynamic viscosity; P, pressure; $h(\alpha, \beta)$, a component of the metric tensor for the bipolar system of coordinates $h(\alpha, \beta) = a/\text{ch}\alpha - \cos\beta$; a, a parameter in the bipolar system of coordinates; β_{in} and β_{out}, bipolar coordinates of entrance and exit respectively; α_0, bipolar coordinate of the roller surface; ξ_H, $= y_H\sqrt{2Rh_0}$, entrance coordinate in Haskell variables; $\xi_{1,max}$, maximally possible exit coordinate in Haskell variables; h_0, half of the minimum gap width; n and s, respectively, normal and tangential directions to an arbitrary surface; Re, Reynolds number; and k_i, k_j, steps along corresponding coordinates in a nonuniform rectangular computation grid.

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